

1. Show, by definition, that

(a)  $\lim_{x \rightarrow 2} \frac{x+5}{x^2-3} = 7$ ,

(b)  $\lim_{x \rightarrow 3^+} \frac{x^2+1}{x-3} = +\infty$  ( $x > 3$ ),

(c)  $\lim_{x \rightarrow 1} \frac{x}{x-1}$  not exist in  $\mathbb{R} \cup \{-\infty, \infty\}$ .

(d)  $\lim_{x \rightarrow x_0} \sqrt{f(x)} = \sqrt{l}$  if  $f$  is a non-negative.

real-valued function on  $\mathbb{R}$  such that  $\lim_{x \rightarrow x_0} f(x) = l \in \mathbb{R}$

and  $x_0 \in \mathbb{R}$ .

2. State (without proof) the following results:

(a) Bolzano-Weierstrass Theorem; (b) Maximum-Minimum Theorem;

(c) Intermediate Value Theorem. (d) Uniform Continuity Theorem.

These theorems are allowed to be used in the following Q3.

3. Prove/Disprove each of the following assertions: either prove it or show it is wrong (the continuity is assumed):

(a)  $h(x) := \frac{1}{x}$  is uniformly continuous on  $(0, \infty)$ ;

(b)  $g(x) := \frac{1}{x}$  is uniformly continuous on  $[3, +\infty)$ ;

(c) Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be continuous and  $\lim_{x \rightarrow \infty} f(x) = l = \lim_{x \rightarrow -\infty} f(x) \in \mathbb{R}$ .

Then  $f$  attains its max. or

$f$  attains its min.